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COMMUTATIVITY OF QUOTIENT RING USING MULTIPLICATIVE GENERALIZED DERIVATION

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Abstract
The current article proves the commutativity of quotient rings R_1/P_1 here R_1 is an arbitrary ring while P_1 denotes the prime ideal of R_1. The strong bonding is established among the behavior of multiplicative generalized derivation, the structure of a rings class, and the left multiplier through the support of some identities containing prime ideals. Moreover, the quotient ring R_1/P_1 characteristics have been determined in case of different situations.

keyword: Generalized derivation, Prime Ideal, Commutativity.

1 Introduction:
In this article R_1 denotes arbitrary Ring with center $Z(R_1)$. Recall that P_1 as an ideal of R_1 is considered prime if $P_1 \neq R_1$, and for $x_1, y_1 \in R_1$, $x_1R_1y_1 \subseteq P_1$, implies that $x_1 \in P_1$ or $y_1 \in P_1$. Therefore, R_1 is termed a prime ring iff (0) is the prime ideal of R_1. R_1 remain 2-torsion free if whenever $2x_1=0$, with $x_1 \in R_1$ implies $x_1=0$. Any pair of elements can be represented $x_1, y_1 \in R_1$, the commutator $[x_1, y_1] = x_1y_1 - y_1x_1$, and anti-commutator $x_1 \circ y_1 = x_1y_1 + y_1x_1$. The basic commutator identities are used at large scale (*i*) $[x_1, y_1z_1] = [x_1, y_1]z_1 + y_1[x_1, z_1]$, (*ii*) $[x_1y_1, z_1] = [x_1, z_1]y_1 + x_1[y_1, z_1]$ and anti-commutator identities (i) $(x_1 \circ y_1z_1) = (x_1 \circ y_1)z_1 - y_1[x_1, z_1]$, (ii) $(x_1y_1 \circ z_1) = (x_1 \circ z_1)y_1 - x_1[y_1, z_1]$. Derivation indicates the additive mapping d: $R_1 \rightarrow R_1$ satisfying $d(x_1y_1) = d(x_1)y_1 + x_1d(y_1)$, for all $x_1, y_1 \in R_1$. A derivation d is inner if $\exists\ a \in R_1$ in the sense that $d_\alpha(x_1) = [a, x_1]$, $\forall\ x_1 \in R_1$. A map H is said to be a left multiplier if it satisfies $H(x_1y_1) = H(x_1)y_1$ for all $x_1, y_1 \in R_1$ [7, 10]. The researchers amplify the former definition by defining a map F: $R_1 \rightarrow R_1$ called a generalized multiplicative derivation if $F(x_1y_1)=F(x_1)y_1 + x_1d(y_1)$, holds for $x_1, y_1 \in R_1$. Furthermore, generalized multiplicative derivation with $d\neq0$, prevails the requirement of multiplier In recent era many researchers have attained commutativity of prime and semi-prime rings by additive mappings, as automorphism derivation, skew derivation and generalized commutative derivation acting on appropriate subsets of rings. First we should keep in mind that $S \subseteq R_1$, a mapping $f: S \rightarrow R_1$, is called centralizing if $[f(x_1), x_1] \in Z(R_1)$, $\forall\ x_1 \in S$. In this particular scenario, where $[f(x_1), x_1] = 0$, $\forall\ x_1 \in S$ and claimed that the mapping is a commutative on S. In [10] Posner justified if a prime ring R_1 admits a nonzero derivation d, and $[d(x_1), x_1] \in Z(R_1)$, $\forall x_1 \in R_1$, then R_1 is commutative. In the last few years many researchers has purified augmented results in [2, 7, 8, 9].
In the literature [2, 3, 5, 7] a number of authors have investigated the commutativity of prime and semi-prime rings that meet specific functional identities that require derivation. In [1] authors studied the following circumstances: (i) $F(x_1y_1) \in Z(R_1)$, (ii) $F([x_1, y_1]) \in Z(R_1)$, (iii) $F(x_1y_1) \pm [x_1, y_1] \in Z(R_1)$ and (iv) $F(x_1y_1) \pm y_1x_1 \in Z(R_1)$ in case of

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non-zero left ideal of semi-prime ring R_1 for all x_1, y_1 R_1. Recently, Dhara et.al. [7] studied the commutativity in semi-prime ring with left ideal and multiplicative generalized derivation. Very recently Faiza Shujat, Shahoor khan and Abu Zaid Ansari study the following conditions: Assume that R_1 is a semi-prime ring, $F : R_1 \to R_1$ be a generalized multiplicative derivation associated to derivation d and the map $H : R_1 \to R_1$ where H be a multiplicative left centralizer such that (i) $F(x_1y_1) \pm H[x_1, y_1] \in Z(R_1)$, (ii) $F[x_1, y_1] \pm H[x_1, y_1] \in Z(R_1)$, (iii) $F(x_1)F(y_1) \pm H[x_1, y_1] \in Z(R_1)$, (iv) $F(x_1)F(y_1) \pm H(x_1y_1) \in Z(R_1)$, (v) $H(x_1)F(y_1) \pm (x_1 \circ y_1) \in Z(R_1)$, (vi) $[F(x_1), H(y_1)] \in Z(R_1)$, and (vii) $(F(x_1) \circ H(y_1)) \in Z(R_1)$. The current article investigates the commutativity of quotient ring by considering some identities containing F and left multiplier.

Main Results

Lemma 1.1. R1 represent a ring containing prime ideal P1. If F is a multiplicative generalized derivation of R1 associated d such that $F(x_1y_1) \in P_1$, for all $x_1, y_1 \in R_1$, then R_1/P_1 is cumulative integral domain.

Proof- *Consider,*

$F(x_1y_1) \in P_1$, for all $x_1, y_1, \in R_1$. (1.1)

Replacing y_1z_1 in the place if y_1 we have,

$F(x_1y_1z_1) = F(x_1y_1)z_1 + x_1y_1d(z_1) \in P_1$, for all $x_1, y_1, z_1 \in R_1$. (1.2)

We get,

$[x_1y_1d(z_1), z_1] \in P_1$, for all $x_1, y_1, z_1 \in R_1$. (1.3)

We obtain,

$[x_1, z_1]y_1d(z_1) \in P1$, for all $x_1, y_1, z_1 \in R_1$. (1.4)

This implies,

$[x_1, z_1]R_1d(z_1) \subseteq P1$, for all $x1, y1, \in R1$. (1.5)

Since $P1$ is a prime ideal, then for all $x1, y1, z1 \in R1$, either $[x_1, z_1] \in P_1$, With $R_1 = \{x_1 \in R_1 / [x_1, z_1] \in P_1$, for all $z_1 \in R_1\}$. In which $R1/P1$ is commutative integral domain.

LEMMA 1.2. *R_1 represent a ring containing prime ideal P1. If F is a multiplicative generalized derivation of R_1 associated with d such that* $F[x_1, y_1] \in P_1$, *for all $x_1, y_1 \in R_1$, then R_1/P_1 is cumulative integral domain.*

Proof*- Assert that,*

$F[x_1, y_1] \in P_1$, for all $x_1, y_1 \in R_1$. (1.11)

Replacing y_1r in the place if y_1 we have,

$F([x_1, y_1])r + [x_1, y_1]d(r) \in P_1$, for all $x_1, y_1, r \in R_1$. (1.12)

This gives,

$[x_1, y_1]d(r) \in P_1$, for all $x_1, y_1, r \in R_1$. (1.13)

We replace y_1z_1 in the place if y_1 we get,

$[x_1, y_1]z_1d(r) \in P_1$, for all $x_1, y_1, z_1, r \in R_1$. (1.14)

This implies,

$[x_1, y_1]R_1d(r) \subseteq P_1$, for all $x_1, y_1, z_1 \in R1$. (1.15)

Since P_1 is a prime ideal, then for all $x_1, y_1, z_1 \in R_1$, $[x_1, y_1] \in P_1$, $R_1 = \{x_1 \in R_1 / [x_1, y_1] \in P_1$, for all $y_1 \in R_1\}$. In which R_1/P_1 is commutative integral domain.

Theorem 1.1. *R_1 represents a ring containing prime ideal P_1. if F be a multiplicative generalized derivation and H be a multiplier such that,* $F[x_1, y_1] \pm H[x_1, y_1] \in P_1$, for all $x_1, y_1 \in R_1$, *then R_1/P_1 is a commutative integral domain.*

Proof. *Assert that,*

$F[x_1, y_1] \pm H[x_1, y_1] \in P_1$, for all $x_1, y_1 \in R_1$. (1.6)

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We replace y_1 by y_1x_1 in (1.6) we have,
$F[x_1, y_1]x_1 + [x_1, y_1]d(x_1) \pm H(x_1y_1)\, x_1 \in P_1$, for all $x_1, y_1 \in R_1$. (1.7)
This gives,
$[x_1, y_1]\, d(x_1) \in P_1$, *for all* $x_1, y_1 \in R_1$. (1.8)
Substituting y_1r *in the place* y_1 *we obtain*
$[x_1, y_1]\, rd\,(x_1) \in P_1$, *for all* $x_1, y_1, r \in R_1$. (1.9)
We have,
$[x_1, y_1]\, R_1d(r) \subseteq P_1$, *for all* $x_1, y_1, r \in R_1$. (1.10)
Since P_1 is a prime ideal, then for all $x_1, y_1, z_1 \in R1$, $[x_1, y_1] \in P_1$, $R_1 = \{x_1 \in R_1/ [x1, y1] \in P1$, for all $y_1 \in R_1\}$. In which R_1/P_1, is a commutative integral domain.

Theorem 1.2. R_1 represents a ring containing prime ideal P_1. if F be a multiplicative generalized derivation and H be a left multiplier such that, $F\,[x_1, y_1] \pm H\,[x_1, y_1] \in P_1$, *for all* $x_1, y_1, \in R_1$, *then* R_1/P_1 is a commutative integral domain.

Proof. If $H=0$, then we have $F\,[x_1, y_1] \in P1$, *for all* $x_1, y_1 \in R1$, hence conclusion follow from lemma (1.2), let us consider $H \neq 0$,
$F[x_1, y_1] \pm H\,[x_1, y_1] \in P_1$, *for* all $x_1, y_1 \in R_1$. (1.16)

Replace y_1 *by* y_1x_1 *in above we get,*
$F[x_1, y_1]x_1 + [x_1, y_1]\, d(x_1) \pm H[x_1, y_1]x_1 \in P_1$, *for all* $x_1, y_1 \in R_1$. (1.17)
We obtain,
$[x_1, y_1]\, d(x_1) \in P_1$, *for all* $x_1, y_1 \in R_1$. *(1.18)*
Put y_1r *in the place* y_1 *we obtain,*
$[x_1, y_1]\, rd(x_1) \in P_1$, *for all* $x_1, y_1, \in R_1$. *(1.19)*
This gives,
$[x_1, y_1]\, R_1d(x_1) \subseteq P_1$, *for all* $x_1, y_1 \in R_1$. *(1.20)*
Since P_1 is a prime ideal, then for all $x_1, y_1, z_1 \in R_1$, $[x_1, y_1] \in P_1$, with $R_1 = \{x_1 \in R_1/ [x_1, y_1] \in P_1$, *for all* $y_1 \in R_1\}$. In which R_1/P_1, is a commutative integral domain.

Theorem 1.3. R_1 represents a ring containing prime ideal P_1. If F be a multiplicative generalized derivation and H be a left multiplier such that $F(x_1)\, F(y_1) \pm H(x_1y_1) \in P_1$. for all $x_1, y_1 \in R_1$ then R_1/P_1 is a commutative integral domain.

Proof. We assume that,
$F(x_1)\, F(y_1) \pm H\,(x_1y_1) \in P_1$, for all $x_1, y_1 \in R_1$. (1.21)
We replace y_1z_1 in the place of y_1 we have,
$F(x_1)F(y_1)z_1+F(x_1)y_1d(z_1)\pm H(x_1y_1)z_1 \in P_1$, for all $x_1, y_1, z_1 \in R_1$. (1.22)
We get,
$[F(x_1)\, y_1d(z_1), z_1] \in P_1$, for all $x_1, y_1, z_1 \in R_1$. (1.23)
This gives,
$F(x_1)\, y_1d(z_1), z_1] \in P_1$. for all $x_1, y_1, z_1 \in R_1$. (1.24)
Replacing r in the place of $d(z_1)$ we obtain,
$F(x_1)\, R_1[r, z_1] \in P_1$. for all $x_1, y_1, z_1 \in R_1$. (1.25)
Then either $F(x_1) \in P_1$ or $[r, z_1] \in P_1$, above given $F(x_1) \in P_1$, for all $x_1 \in P_1$,
we replace x_1y_1 in the place of x_1 we have,
$F(x_1)y_1+x_1\, d(y_1) \in P_1$, for all $x_1, y_1 \in R_1$. (1.26)
We arrive at,
$[x_1d(y_1), y_1] \in P_1$ for all $x_1, y_1, \in R_1$. (1.27)
Replace x_1r in the place x_1 this gives,
$[x_1, y_1]rd(y_1) \in P_1$. for all $x_1, y_1 \in R_1$. (1.28)

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We have,
$[x_1, y_1] R_1 d(y_1) \subseteq P_1$, for all $x_1, y_1 \in R_1$. (1.29)
Since P_1 is a prime ideal then for all $x_1, y_1, z_1 \in R_1$, either $[x_1, y_1] \in P_1$.
with $M = \{x_1 \in R_1 / [x_1, y_1] \in P_1$, for all $y_1 \in R_1\}$. In which R_1/P_1, is commutative integral domain.

Theorem 1.4. R_1 represents a ring containing prime ideal P_1. F be an multiplicative generalized derivation of R_1 associated d. H be left multiplier such that $F(x_1)F(y_1) \pm H,[x_1, y_1] \in P_1$, for all x_1, y1ϵ R_1 then R_1/P_1 is commutative integral domain.

Proof. Assert that,
$F(x_1)F(y_1) \pm H[x_1, y_1] \epsilon P_1$. for all $x_1, y_1 \in R_1$. (1.30)
We put y_1x_1 in the place y_1 we obtain,
$F(x_1)F(y_1)x_1 + F(x_1)y_1 d(x_1) \pm H([x_1,y_1]) x_1 \in P_1$, for all $x_1, y_1 \in R_1$. (1.31)
This implies,
$[F(x_1)y_1 d(x_1), x_1] \in P_1$, for all $x_1, y_1 \in R_1$. (1.32)
We have,
$F(x_1)y_1[d(x_1), x_1] \in P_1$ for all $x_1, y_1 \in R_1$. (1.33)
Using theorem (1.5), R_1/P_1 is commutative integral domain.

Theorem 1.5. R_1 represents a ring containing prime ideal P_1. F be a multiplicative generalized derivation of R_1 associated d. H be left multiplier such that
$H(x_1) F(y_1) \pm (x_1 \text{ o } y_1) \epsilon P_1$, for all $x_1, y_1 \in R_1$, then

(i)$H \subseteq P_1$, (ii) R_1/P_1 is commutative integral domain.

Proof. Assert that,
$H(x_1)F(y_1) \pm (x_1 \text{ o } y_1) \in P_1$, for all $x_1, y_1 \in R_1$. (1.34)
Replacing y_1x_1 in the place of y_1 we obtain,
$H(x_1)F(y_1)x_1 + H(x_1)y_1 d(x_1) \pm (x_1 \text{ o } y_1)x_1 \pm y_1[x_1, x_1] \in P_1$, for all $x_1, y_1 \in R_1$. (1.35)
We get,
$[H(x_1)y_1 d(x_1), x_1] \in P_1$, for all $x_1, y_1 \in R_1$. (1.36)
This gives,
$H(x_1)y_1[d(x_1), x_1] \in P_1$, for all $x_1, y_1 \in R_1$. (1.37)
We arrive at,
$H(x_1) R_1[d(x_1),x_1] \subseteq P_1$. for all $x_1, y_1 \in R_1$. (1.38)
Since P_1 is a prime ideal then either $H \subseteq P_1$, or $[d(x_1), x_1] \epsilon P_1$, this implies that $M = R_1 \cup R_2$ with $R_1 = \{x_1 \in R_1/[d(x_1), x_1] \in P_1\}$ and $R_2 = \{x_1 \epsilon R_2/H(x_1) \in P_1\}$, Since a group cannot be union of its subgroup then $M = R_1$ in which case R_1/P_1 is commutative integral domain. In the other case $H(R_2) \subseteq P_1$.

Theorem 1.6. R_1 represents a ring containing prime ideal P_1. F be a multiplicative generalized derivation of R_1 associated d. H be left multiplier such that
$[F(x_1), H(y_1)] \in P_1$, for all x_1. $y_1 \in R_1$, then
(i)$H \subseteq P_1$, (ii) R_1/P_1 is commutative integral domain.

Proof. Assert that,
$[F(x_1), H(y_1)] \in P_1$, for all $x_1, y_1 \in R_1$. (1.39)
Replacing y_1z_1 in the place of y_1 we have,
$[F(x_1), H(y_1)z_1] \in P_1$, for all $x_1, y_1, z_1 \in R_1$. (1.40)
We get
$H(y_1) [F(x_1),z_1] \in P_1$, for all $x_1, y_1, z_1 \in R_1$ (1.41)
We put y_1r in the place of y_1 we have,

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$H(y_1)r[F(x_1), z_1] \in P_1$. for all $x_1, y_1, z_1, r \in R_2$. (1.42)
We arrive at,
$H(y_1) R_1[F(x_1), z_1] \subseteq P_1$, for all $x_1, y_1, z_1 \in R_1$. (1.43)
Since P_1 is a prime ideal so either $H(R_1) \subseteq P_1$, or $(F(x_1), z_1] \in P_1$, which implies that $M = R_1 \cup R_2$, with $R_1 = \{x_1 \in R_1/H(x_1) \in P_1\}$ and $R_2 = \{x_1 \in R_1/[F(x_1), z_1]$ for all $z_1 \in R_1\}$,
since a group cannot be union of its subgroups then $M= R_1$ in which case $H\subseteq P_1$, and $R_2= \{x_1 \in R_1/[F(x_1), z_1]$ forall $z_1 \in R_1\}$. Put x_1r in the place of x_1 we have,
$[x_1d(r), z_1] \in P_1$. for all x_1. r. $y_1 \in R_1$. (1.44)
Replacing x_1y_1 in the place of x_1 we have $[x_1, z_1]y_1d(r) \in P_1$, then by theorem (1.3) we have R_1/P_1 is commutative integral domain.
Theorem 1.7. R_1 represents a ring containing prime ideal P_1. F be a multiplicative generalized derivation of R_1 associated d. H be left multiplier such that $F(x_1)$ o $H(y_1)$ P_1, for all $x_1, y_1 \in R_1$, then
(i)$H\subseteq P_1$, (ii) R_1/P_1 is commutative integral domain.
Proof. Assert that,
$F(x_1)$ o $H(y_1) \in P_1$, for all $x_1, y_1 \in R_1$. (1.45)
Replacing in the y_1z_1 place of y_1 we have,
$F(x_1)$ o $H(y_1) z_1 \in P_1$, for all $x_1, y_1 \in R_1$. (1.46)
This implies,
$H(y_1) [F(x_1), z_1] \in P_1$. for all $x_1, y_1.z_1 \in R_1$. (1.47)
Using (1.6) theorem that,
(i)$H\subseteq P$, (ii) R_1/P_1 is commutative integral domain.

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