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BIPARTITE THROUGH PRESCRIBED MEDIAN AND ANTIMEDIAN OF A COMMUTATIVE RING WITH RESPECT TO AN IDEAL

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ABSTRACT:

There are plenty of ways of partners with arithmetical constructions. Some of them to make reference to are bipartite from gatherings, median and anti – median from commutative rings with reference to an ideal. Partnering a with median and anti – median of a commutative ring was presented by Beck in 1988. Similarly Beck has researched the exchange between the ring theoretic properties of a commutative ring and related median and anti – median. Further Anderson and Badawi presented the idea absolute of commutative rings with median and anti – median in the year 2008. The absolute of a commutative ring *R* is the undirected with *R* as the vertex set and two particular vertices in *R* are nearby if and provided that their total is a median and anti – median of*R*. As of late Anderson and Badawi presented and concentrated on the summed up all out the of commutative rings concerning the multiplicatively prime subset *H* of *R*. The summed up complete of a commutative ring is the undirected with all components of *R* as vertices, and for two unmistakable vertices in *R* are nearby if and provided that their total is in *H*. In this article, an endeavor has been made to learn about in hypothetical properties and different control boundaries of summed up absolute of commutative rings of median and anti – median and its supplement.

Keywords: median, anti - median, commutative ring, ideal.

2010 Mathematics Subject Classification: 05C12.

1. Introduction

Let $G = (V, E)$ be a on n vertices with vertex set *V* and edge set *E*. A is bipartite if its vertex set can be partitioned into two nonempty subsets *X* and *Y* such that each edge of *G* has one end in *X* and the other in *Y*, and a is *k*-partite if its vertex set can be partitioned into *k* nonempty subsets such that no edge in G has its both ends in the same subset. Degree of a vertex v, $d(v)$, is the number vertices adjacent to v and by $N(v)$ we denote the neighbor set of v. The smallest and largest degrees of vertices in *G* are respectively denoted by $\delta(G)$ and $\Delta(G)$.

Given a *G* the issue of tracking down a *H* to such an extent that $M(H) \cong G$ is alluded to as the median issue. In [6], it is shown that any $G = (V, E)$ is the median of some associated . In [3] the thought of against median of a was presented and demonstrated that each is the counter median of some . The issue of concurrent inserting of median and hostile to medianis examined in [1]. Another development, which sums up every one of the recently referenced developments, can be found in [5].

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The median vertices have the base normal distance in *a* and subsequently the median issue is huge among the improvement issues including the position of organization servers. Nonetheless, the median developments for general can't be straightforwardly applied to a huge number as their fundamental has a place with various classes of . It tends to be seen that are bipartite to basic s of a huge number. For instance, the vast majority of the examinations in network networks are finished utilizing inclination networks [4] and they are displayed utilizing bipartite .

It is notable that the median of a tree is a vertex or an edge. This administrator was additionally read up for certain classes of in [7] and [8]. In this paper we show that any bipartite is the median of another bipartite . With an alternate development, we show that the comparative outcome additionally hold for k-partite. The undifferentiated from results for against median issue on these classesare additionally acquired. Since any is a k-partite, for some *k*, these developments can be applied overall. For any remaining fundamental ideas and documentations not referenced in this paper we allude to [2].

In variation to the concept of zero divisor, few authors [8] introduced the total of a commutative ring. Let *R*be a commutative ring with *Nil(R)* its ideal of nilpotent elements, *Z(R)* its set of zero-divisors, and *Reg(R)* its set of regular elements. The total of *R,* denoted by *T (R),* is the undirected with all elements of *R*as vertices, and for distinct $x, y \in R$, the vertices *x* and *y*are adjacent if and only if $x + y \in Z(R)$. Also they introduced the three induce subs *Nil (R) Z (R)* and *Reg (R)* of *T (R)* with vertices *Nil(R), Z(R),*and *Reg(R).*

A graph wherein each sets of particular vertices is joined by an edge is known as a total graph. We use K_n for the total graph with *n* vertices. A *r*-partite graph is a graph whose vertex set can be divided into *r* subsetsso that no edge has the two vertices in any one subset. A total I-partite graph is one in which every vertex is joined to each vertex that isn't in a similar subsetas the given vertex. The total bipartite (i.e., complete 2-partite) graph is signified by $K_{m,n}$ where the arrangement of segment has sizes *m* and *n*. The circumference of a graph*G* is the length of a most limited cycle in *G* and is meant by bigness *(G)*. We characterize a shading of a graph*G* to be a task of tones (components of some set) to the vertices of *G*, one tone to every vertex, so nearby vertices are doled out unmistakable tones. In the event that *n* tones are utilized, the shading is alluded to as a *n*-shading. On the off chance that there exists a *n*-shading of a graph*G, G* is called *n*-colorable. The base *n* for which a graph*G* is n-colorable is known as thechromatic number of *G*, and is indicated by $\chi(G)$. A club of a graph is a maximal complete sub and the quantity of vertices in the biggest inner circle of graph*G*, signified by $\omega(G)$, is known as the faction number of *G*. Clearly$\chi(G) \geq \omega(G)$ for general graph*G.*

Assume that *S* is a commutative semigroup with nothing. For ideal hypothesis in commutative semigroup, we allude to the overview of median and anti – median [3] (additionally see [2]). Here we simply review a portion of the ideas. A non-void subset *I* of *S* is called ideal if $xS \subseteq I$ for any $x \in I$. An optimal *p* of a

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commutativesemigroup is known as an excellent ideal of *S* if for any two component *x, y* ∈ *S, xy*∈ *p* infers *x* ∈ *p* or *y* ∈ *p*. Let *Z(S)* be its arrangement of zero-divisors of *S*. All together that *Γ(S)* be non void, we generally expect *S* generally contains somewhere around one nonzero zero divisor. In [14] we can view that *Γ(S)* (as in the ring case) is generally associated, and the breadth of *Γ(S)* ≤ *3*. In the event that *Γ(S)* has a cycle bigness *(Γ(S))* ≤ *4*. They additionally show that the quantity of insignificant beliefs of *S* gives a lower bound to the coterie number of *S*. In [26] authors concentrated on a graph Γ(S) where the vertex set of this chart is *Z(S)* ∗ and forparticular components *x, y* ∈ *Z(S)* ∗, in the event that *xSy = 0*, there is an edge interfacing *x* and *y.* Note that *Γ(S)* is a subgraph of *Γ(S).* As oflate, several authors concentrated on additional the graph*Γ(S)* and its augmentation to a simplicial complex, cf. [13]. Obviously for any superb ideal *p* in the event that *x* and *y* are nearby in *Γ(S), x* ∈ *p* or *y* ∈ *p*. So, for each superb ideal *p* and each edge *e,* one of the end points of *e* has a place with *p,*

Building graphs from commutative rings was started by Ivan Beck through his work on zero-divisor charts and from that point a few graphs developments were made by a few creators. Through the developmentof charts from commutative rings, exchange between mathematical properties of commutative rings and graphs hypothetical properties of determined charts are contemplated. A portion of the charts characterized out of gatherings are Cayley graphs from bunches [25], non-commutating chart of a gathering [2], power chart of a limited gathering [29]. A chart is characterized out of non no divisors of a ring and is called zero-divisor graphs of a ring [12]. Intriguing varieties are likewise characterized like all out graphs[8], unit charts [15] and comaximal charts [33] related with rings. Additionally, charts are characterized out of standards of a ring, to be specific obliterating ideal graphs of a ring [23], convergence chart of beliefs of rings [30, 31] and so forth.

Connecting agraphs with zero-divisors of a commutative ring was presented [24] in 1988, where the creator discussed shading of such graphs. Subsequent to presenting zero-divisor graphs, I. Beck made a guess that the faction number and chromatic number of the zero-divisor graphs are equivalent. In 1993, few authors settled Beck's guess in negative by giving a counter model [11]. Additionally, they have explored the interchange between the ring hypothetical properties of a commutative ring and graphs hypothetical properties of the zero-divisor chart. The definition alongside name for zero-divisor chart (R) was first presented in 1999, subsequent to altering the meaning of D. D. Anderson [11, 12].

Example of a zero-divisor graph for $R = Z_2 X \frac{Z_2(x)}{<x^2>}$ is shown in fig 1.1

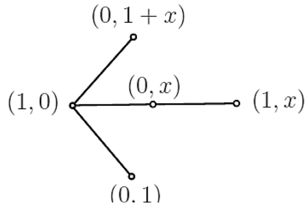

Figure 1.1: $R = Z_2 X \frac{Z_2(x)}{<x^2>}$

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Let us collect some basic definitions and results on commutative rings:

Definition 1.1.[2] A ring (R,+, ·) is a nonempty set R together with binary operations '+'and '·'defined on R, which satisfy the following conditions:

(i) $(R,+)$ is an abelian group

(ii) $a \cdot (b \cdot c) = (a \cdot b) \cdot c, \forall a, b, c \in R$

(iii) $a \cdot (b + c) = a \cdot b + a \cdot c, \forall a, b, c \in R$

(iv) $(a + b) \cdot c = a \cdot c + b \cdot c, \forall a, b, c \in R$.

Definition 1.2. [9] A ring *R*is called commutative if for every $a, b \in R$,

$\exists a \cdot b = b \cdot a$.

Definition 1.3. [10] Let *R*be a ring. An element $e \in R$is called an identity element if $ea= ae = a \; \forall a \in R$. The identity element of a ring *R*is denoted by '*1*'.

Definition 1.4. [16] Let *R*be a ring with identity. An element $u \in R$is called a unit element if there exists $v \in R$such that $uv= 1 = vu$and the inverse of u is often denoted by $u-1$. The collection of all units in *R*is denoted by $U(R)$ or $R\times$.It is easy to check that $U(R)$ is a group under multiplication and is called multiplicative group of *R*.

Definition 1.5. [17] A ring *R*with identity is called a division ring if every nonzero element of *R*is a unit. A commutative division ring *R*is called a field.

Definition 1.6. [18] An element $x \in R$is said to be a zero-divisor if thereexists $0 \neq y \in R$ $\exists \; xy = 0$ where *0* is the additive identity. The set of all zero-divisors in *R*is denoted by $Z(R)$.

Definition 1.7. [19] An ideal *P*of a ring *R*is called a prime ideal if $P \neq R$and $\forall a, b \in R$, $ab \in P$implies $a \in P$ or $b \in P$.

Definition 1.8. [20] A commutative ring *R*is called an integral domain if*R*has no non-zero zero-divisors.

Definition 1.9. [21] Let *R*be a ring. The characteristic of *R*is the leastpositive integer *n*such that $na= 0 \forall a \in R$. If no such positiveinteger exists, then *R*is said to be of characteristic zero.

Let us collect some basic definitions and results on s:

Definition 1.9. [5] Given a bipartite *G* of n vertices, there exists a connected bipartite *H′* such that *G* is an induced sub of *H′* and all the vertices of *G* in *H′* have equal status in *H′.*

Definition 1.10. [22] Given a bipartite *G* there exists a bipartite *H* such that

$M(H) \cong G$.

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Definition 1.11. [27] Given a bipartite *G* there exists a bipartite *H* such that

AM(H) ≅ *G*

Definition 1.12. [28] Given a *k*-partite *G* there exists a *k*-partite *H* such that

M(H) ≅ *G.*

Definition 1.13 [32] Given a *k*-partite *G* there exists a *k*-partite H′ such that

AM(H′) ≅ *G.*

2.0 Main Results

Entrenching Median and Anti - medianconceptions.

Theorem2.1. For a determinate commutative semigroup *S*, the set *V(φ(Γ(S)))∪{0}* is an median of *S*.

Proof. Let *x* ∊ *V(φ (Γ(S))),* and *r* ∊ *S*. Suppose that *rx*≠ 0.

Then *e(rx) = max{d (u, rx)|u* ∊ *V(G)}* ≤ *max{d (u, x)|u* ∊ *V(G)} = e(x).*

Thus *e(rx)* = *e(x)*.

Hence,*rx*∊ *V(φ (Γ(S))) ∪ {0}.*

Remark 2.1. A subgraph *H* of a graph*G* is a crossing subgraph of *G* if *V(H) = V(G)*. On the off chance that *U* is a bunch of edges of a graph*G, G \ U* is the crossing subgraph of *G* acquired by erasing the edges in *U* from *E(G).* A subset *U* of the edge set of an associated chart *G* is an edge cutset of *G* if *G \ U* is separated. An edge cutset of *G* is negligible assuming no appropriate subset of *U* is edge cutset. Assuming *e* is an edge of *G,* with the end goal that *G \ {e}* is detached, then *e* is known as an extension. Note that on the off chance that *U* is an insignificant edge cutset, *G \ U* has precisely two associated median parts.

Corollary 2.1. Let *T* be the minimal edge cutset of *Γ(S),* and *G1, G2* are two median parts of *G \ T*. Then the following hold.

(i) For any *i = 1, 2, (V(Gi)* ∩ *V(T))* ∪ *{0}* is ideal of *S* provided $G_i$has at least two vertices.

(ii) *V(T)* ∪ *{0}* is an ideal if G_1 or G_2 has only one vertex. A commutative semigroup is called reduced if for any *x* ∊ *S,* x_n = *0* implies *x* = *0*. The annihilator of *x* ∊ *S* is denoted by *Ann (x)* and it is defined as *Ann (x)* = *{a* ∊S|*ax* = *0*}. In Satyanarayana gave some characterization of *s.*

Theorem 2.2. Let S be a commutative and reduced semigroup in which Γ(S) does not contain a median and anti – median clique. Then S satisfies the ascending chain conditions(a.c.c) on annihilators.

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Proof. Suppose that *Ann* x_1< *Ann* x_2<$\cdots < Ann\ x_n$be acumulative chain of ideals. For each$i \geq 2$, select $a_1 \in Ann x_1 \backslash Ann x_{i-1}$. Then every$a_n$isnonzero, for $= 2,3,\cdots$. Also $y_i y_j$for any $i \neq j$. Since S is a commutativeand condensed semigroup, we have $y_i \neq y_j$ when $i \neq j$. Thus,one can obtain anmedian and anti – median in*S*. This is a contradiction and so the assertion holds.

Theorem 2.3. Let S be a commutative rings of median and anti – median. Then the subsequent results hold:

(i) If |*Ass (S)*| ≥ *3* and ∅ = *Ann (x),*χ = *Ann (y)* are two distinct elements of Ass (S), then *xy* = *0*.

(ii) If |*Ass (S)*| ≥ *3*, then girth *(Γ(S)) = 4.*

(iii) If |*Ass (S)*| ≥ *6*, then *Γ(S)* is not planar (A graph *G* is planar if it can be drawn in the plane in such a way that no two edges meet except at vertex with which they are both incident).

Proof. (i). We can assume that there exists r ∈∅ \ χ. Then *rx* = *0* and *so rSx* = *0* ∈∅. Since χ is a prime ideal, *x* ∈$\chi$and hence *xy* = *0*.

(ii). Let $\emptyset_1$ = *Ann* (x_1), $\emptyset_2$= *Ann* (x_2),$\emptyset_3$= Ann (x_3) and $\emptyset_4$= Ann (x_4) belong to *Ass (S).* Then $x_1 - x_2 - x_3 - x_4 - x_1$ is a cycle of length 4.

(iii). Since |*Ass (S)*| ≥ *6*, $k_5$is a subgraph of *Γ(S),* and hence by Kuratowski's Theorem *Γ(S)* is not planar.

Corollary 2.2. Let $\emptyset_1$ = *Ann* (x_1), $\emptyset_2$= *Ann* (x_2),$\emptyset_3$= Ann (x_3) and $\emptyset_4$= Ann (x_4), $\cdots \emptyset_n$= Ann (x_n) belong to *Ass (S)*with reference to commutative rings of median and anti – median. Then $x_1 - x_2 - x_3 - x_4 - \cdots x_n$ is a cycle of length *n*.

Remark 2.2. Let *S* be a commutative semigroup and let *Anna* be a maximalelement of$\{Annx : 0 \neq x \in S\}$. Then *Ann a* is a prime ideal.

Theorem 2.4. Let *S* be a commutative semigroup, then the median graph of a bipartite graphis induced by the vertices of maximum degree in G.

Proof.*S* be a commutative semigroup and *G* is a bopartite median graph, thus *d(v, u)* < *2* for any pair of vertices*u, v* of G. Let the degree of *v* in *G* be *d.* Then, these *d* vertices are at a distance1 from *v.* So, there are *p - 1 - d* vertices u in G such that *d(v, u)* = 2 and*D(v) = d + 2(p - 1 - d) = 2(p - 1) - d.* Hence the vertices in G such that *D(v)*is minimum are those for which the degree is maximum. Hence the proof.

Remark 2.3 The median graph of a bipartite graph is also a bipartitegraph.

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