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# RELATION BETWEEN GRAPH AND LITACT GRAPH USING SPEARMAN'S CORRELATION COEFFICIENT

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**Abstract:**

Spearman's correlation coefficient formula has been used as a conglomerate or quantities the direction and strength of alliance between two classified variables widely used in various fields of science and research. The survey of the Spearman correlation coefficient correlated with graph theory by way of adjacency matrix for standard graphs in the Spearman correlation calculations in the case of the binary variable. The outcomes of adjacency matrix for graphs and litact graphs can be used for a non-monotonic relationship between two variables, in lieu of the direction and strength of a linear affinity among the two variables, are determined by Pearson's correlation.

***Keywords:*** Adjacency matrix, Graph, Litact graph, Rank of a matrix, Spearman's Correlation Coefficient, Standard graphs, t-distribution,

## 1. INTRODUCTION

A strong affinity among two or more variables was discussed in the study of correlation [1]. Widely used techniques by researchers is correlation which is one of the statistical analysis techniques because researchers are often interested in what is happening and are trying to connect with them and also apply. The closeness of the affinity among two or more variables can be considered by taking the number of correlation numbers commonly called the correlation of the coefficient. Nonparametric statistical methods are frequently termed as free distribution methods because their testing statistical models do not specify specific conditions as for the distribution of population parameters [2]. The hypothesis that the observations should be independent and that the variables studied should be essentially continuous was specified by the non-parametric statistical test only. Nonparametric statistical tests are at times touch on to as ranking tests since these nonparametric methods are only available for ranking scores kind of accurate scores in the sense of deficiency. One way to measure the nonparametric correlation coefficient is Spearman's rank correlation coefficient. Spearman's rank correlation coefficient was first proposed by Charles Spearman and it is denoted by $\rho$. It can have two senses. First, a perfect Spearman correlation results when X and Y are related to a monotonic function. Compare this with the Pearson correlation, which gives an absolute value only when X and Y are related by a linear function. It evaluates how well the affinity among two variables can be delineated applying a monotonic function. If the data are ordered or use ranks, use Spearman's rank correlation coefficient to find a affinity among two or more variables and test the hypothesis using some appropriate statistical inference test. The

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closeness between two variables can be expressed as the maximum score that can be obtained by the ratio (comparison) of the actual scores +1 and -1. +1 is assigned to a naturally arranged pair and -1 is assigned to a pair that is not naturally composed. +1 indicates a strong positive correlation where as -1 indicates a strong negative correlation between the variables. Spearman's coefficient correlation is suitable for both continuous and discrete ordinal variables. In 1736, Leonard Euler introduced graph theory for the first time, when he demonstrated the chance of four domains connected by seven bridges over the Pregel River in Konigsberg, Russia, each passing over once, and then return to the earliest location [4]. The Konigsberg bridge problem can be represented by identifying four regions as vertices and seven bridges as edges connecting the agnate pairs of vertices. Calculations using graph theory are done by standard graphs, the vertices being each object of study. Write adjacency matrix for Standard graphs and litact graphs of standard graphs. Rank can be achieved by using echelon form for adjacency matrix.

## 2. MATERIALS AND METHODS

The steps to approach Spearman's rank correlation coefficient $\rho$ are through graph theory methods:

i) All graphs are simple, finite and undirected graphs. Consider standard graphs such as cycle graphs, path graphs, wheel graphs, star graphs, complete graphs and complete bipartite graphs.

ii) Establish adjacency matrix for all graphs and litact graphs we considered.

iii) How to form Adjacency matrix:

a) For a simple graph with vertex set $V = \{v_1, v_2, \dots \dots . v_n\}$ the adjacency matrix of order $n \times n$ is

$$Aij = \begin{cases} 1, \text{If there is an edge between vertex } \mathrm{v_i} \text{to vertex } \mathrm{v_j}, \\ \quad 0, \text{otherwise} \end{cases}$$

b) In such matrix the diagonal elements are all zero because edges (self-loops) from vertices to themselves are not allowed in simple graphs.

iv) Calculating Rank of all adjacency matrices formed.

v) Calculate spearman's Rank correlation coefficient.

vi) By t distribution, we inter relate graphs and litact graphs.

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## 3. DEFINITIONS

### Definition 3.1: Graph

A mathematical structure formed with a set of points or nodes joined by a finite number of lines or curves is called as a *graph*. The set of nodes or points is called a vertex set denoted by $V$ and the set of lines or curves is called edge set denoted by $E$. In general, a graph is represented by $G = (V, E)$.

**Definition 3.2:** A vertex set in a litact graph is $E \cup C$ where E is edge set and C is cut vertex set of $G$ where two vertices are adjacent if and only if the corresponding edges and the cut vertices are adjacent or incident on $G$ is called a litact graph $m(G)$ of $G$.

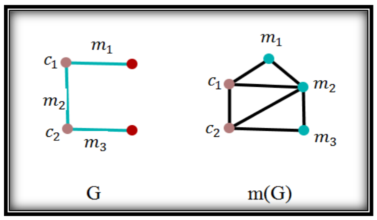


**Fig 1:** $\boldsymbol{Graph\ G\ and\ Litact\ graph\ m(G)}$

### Definition 3.3 Spearman's Rank Correlation Coefficient:

The **Spearman's Rank Correlation Coefficient** is the non-parametric statistical measure used to study the strength of association between the two ranked variables. This method is applied to the ordinal set of numbers, which can be arranged in order, i.e. one after the other so that ranks can be given to each.

In the rank correlation coefficient method, the ranks are given to each individual on the basis of its quality or quantity, such as ranking starts from position 1$^{st}$ and goes till N$^{th}$ position for the one ranked last in the group.

The rank correlation coefficient is calculated by the formula $\rho = \frac{(1-6\Sigma D^2)}{N(N^2-1)}$

The rank correlation coefficient when ranks are repeated are calculated by the formula is

$$\rho = 1 - \frac{6\{\Sigma D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots \ldots\ldots\}}{N(N^2-1)}$$ Where m = number of items whose ranks are common.

**The value of $\rho$ lies between ±1 such as:**

$\rho$ =+1, the order of ranks is exactly the same, moving in the same direction.
$\rho$ =-1, the order of ranks is exactly the same, but is in opposite directions.
$\rho$ =0, no relationship between ranks.

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## 4. RESULTS AND DISCUSSION

### 4.1 AN ADJACENCY MATRIX TO DETERMINE SPEARMAN'S RANK CORRELATION COEFFICIENT:

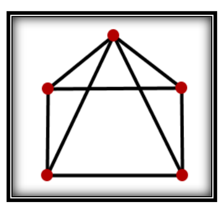
The graph used to calculate Spearman's rank correlation coefficient is an undirected graph since in the given data each consideration is sequentially paired by the study of vertices. In Spearman's rank correlation method, the noticed vertices are associated with every next vertex i.e, $(v_i, v_j)$ and since in the study every pair of vertices must be connected or in other words there is only one edge between the vertices $(v_i, v_j)$ of each pair.

From a vertex $v_i$ to a vertex $v_j$ there is a line called edge. In view of the fact that $(v_i, v_j)$ is connected only once so that there is no parallel line and no self-loops then the graph is simple, undirected graphs. There is exactly one edge between every vertex pair then the graphs form used in to find ranks and spearman's rank correlation coefficient.

Graphs formed adjacency matrix which are symmetric and square. The corresponding adjacency matrix is

given by $Aij = \begin{cases} 1, \text{if there is an edge between vertex } v_i \text{ to vertex } v_j, \\ 0, \text{otherwise} \end{cases}$

**Example 1:**



Adjacency matrix for example 1 is:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

By using the above conditions we formed adjacency matrix for both graphs and litact graphs, we find the ranks of graphs and litact graphs by echelon form using software called matlab. Approach of spearman's rank correlation coefficient by way of Graph theory can be found by using:

$$\rho = 1 - \frac{6\{\Sigma D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots \ldots\ldots\}}{N(N^2 - 1)}$$

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Where, $D^2$ is sum of squares of the differences of two ranks.

N – Number of observations

$m_1, m_2, \dots\dots$ are number of ranks repeated.

$\rho$ is rank correlation coefficient.

Additionally, we heightened null and alternative hypotheses, which can be tested by hypothesis testing using the following formula: $t = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

Where, $\bar{x} = \frac{\Sigma x_i}{n}$

$\bar{y} = \frac{\Sigma y_i}{n}$ and

$S^2 = \frac{\Sigma(x_i-\bar{x})^2+\Sigma(y_i-\bar{y})^2}{n_1+n_2-2}$

**4.2 FURTHER TESTING:**

1. Estimate the null and alternate hypothesis

$H_0$: There is a similarity among graphs and Litact graphs

$H_1$: There is no similarity among graphs and Litact graphs

2. Criteria of decision making

$H_0$: Accepted and $H_1$ is rejected

$H_1$: Accepted and $H_0$ is rejected

3. Level of Significance $\alpha$ at 5%

4. Critical Value at $\alpha$=0.05

With degrees of freedom $v = n_1 + n_2 - 2$

= 23+23 – 2

= 44

$t_\alpha = 1.671$

5. Test statistics: $t = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

Where $\bar{x} = \frac{\Sigma x_i}{n} = \frac{80}{23} = 3.478$ , Assumed mean $A_1 = 3$

$\bar{y} = \frac{\Sigma y_i}{n} = \frac{119}{23} = 5.17,$ Assumed mean $A_2 = 5$ and

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$$S^2 = \frac{\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2}{n_1 + n_2 - 2} = \frac{65+144}{44} = 4.75$$

$$S = \sqrt{4.75} = 2.1794$$

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{3-5}{2.1794\sqrt{\frac{1}{23} + \frac{1}{23}}} = \frac{-2}{2.1794(0.2947)} = \frac{-2}{0.6422} = -3.1142$$

$$|t| = 3.1142$$

$$t > t_\alpha$$

We reject $H_{0.}$

Then there is no similarity between graphs and litact graphs

## 4. CONCLUSION

The ranks of the graphs and the litact graphs can be obtained by accumulating all the arcs of the standard graph, and the test hypothesis can be obtained by the adjacency matrix of the standard graphs and litact graph of standard graphs. Therefore, from the results discussed above, it can be concluded that an undirected graph can be used to determine the value of the Spearman rank correlation coefficient.

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